*JBG-003-1161004*

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Seat No. ______

## M. Sc. (Sem. I) (CBCS) Examination

**December - 2019**

## Mathematics : CMT - 1004

***(Theory of Ordinary Differential Equation)***

**Faculty Code : 003**
**Subject Code : 1161004**

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

**Instructions :** (1) Answer all questions.
(2) The figures on the right hand side indicate the marks allotted to the questions.

**1** Answer any seven : **7×2=14**

(a) Define Degree of a differential equation and linear differential equation with examples.

(b) Show that $\Gamma z = (z-1)\Gamma(z-1)$.

(c) State Variation of constant formulae for scalar linear second order non-homogenous differential equation.

(d) Define Laplace Transform of a function in $\mathcal{K}$ and Show that it converges absolutely.

(e) Prove that $\exp\left(T^{-1}AT\right) = T^{-1}\exp(A)T$.

(f) State change of scale property and $1^{st}$ shifting property in Laplace Transform.

(g) Find general solution of $y^4 + 16y = 0$ on $\mathbb{R}$.

(h) State the First fundamental theorem of calculus.

(i) State the Abel's formula.

(j) Locate and classify the singularities of

$$t^2 y'' + ty' + \left(t^2 - n^2\right)y = 0.$$

**2** Answer any **two** : **2×7=14**

(a) Let $A$ be a constant $2 \times 2$ complex matrix then prove that there exists a constant $2 \times 2$ non-singular matrix $T$ such that $T^{-1}AT$ has the following forms :

(a) $\begin{bmatrix} \lambda & 0 \\ 0 & \mu \end{bmatrix}$ and (b) $\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$.

(b) Let $A$ be a constant $n \times n$ real matrix. Let $g(t)$ be a continuous $n \times 1$ matrix on $(-\infty, \infty)$ and $n_0 \in R^n$ then prove that the unique solution of IVP :

$y' = A(t)y + g(t); y(t_0) = n_0$ is

$$u(t) = \exp(t - t_0) A \cdot n_0 + \int_{t_0}^{t} (\exp(t-s)A) \cdot g(s)\, ds; \forall t \in (-\infty, \infty)$$

Further find the solution of the IVP :

$$y' = \begin{bmatrix} 3 & 5 \\ -5 & 3 \end{bmatrix} y + \begin{pmatrix} e^{-t} \\ 0 \end{pmatrix}; y(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

(c) Find the Eigen values and the corresponding Eigen vector of matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -1 & -6 \end{bmatrix}$.

**3** All are compulsory : **2×7=14**

(1) State and prove Variation of constant formulae for scalar linear 1st order non-homogenous differential equation.

(2) Find Fundamental Matrix of $y' = A(t)y$ on $(-\infty, \infty)$ where $A(t) = \begin{bmatrix} 2 & -3 & 3 \\ 4 & -5 & 3 \\ 4 & -4 & 2 \end{bmatrix} \forall t \in (-\infty, \infty)$ and Find $\exp(tA); \forall t \in (-\infty, \infty)$.

**OR**

**3** All are compulsory : **2×7=14**

(1) Find the solution of the I.V.P :

$$y' = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} y + \begin{pmatrix} 0 \\ e^{-2t} \end{pmatrix}; y(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \text{ on } R.$$

(2) Prove that Eigen vectors corresponding to the distinct Eigen values of $n * n$ matrix are linearly independent in $\mathbb{R}^n$ or $\mathbb{C}^n$.

4 Answer any two : **2×7=14**

(1) Justify weather the Legendre's equation

$\left(1-t^2\right)y''-2ty'+n(n+1)y=0$; (where $n$ is constant)

has a solution or not.

(2) Prove that if $a_0(t)$, $a_1(t)$, $a_2(t)$ which are analytic at $t_0$ and $t_0$ is a regular singular point of $a_0(t)y''+a_1(t)y'+a_2(t)y=0$ then given equation can be written in the form $\left(t-t_0\right)^2 y''+\left(t-t_0\right)\alpha(t)y'+\beta(t)y=0$ for some functions $\alpha(t)$ and $\beta(t)$ which are analytic at $t_0$ and not all $\alpha\left(t_0\right), \beta\left(t_0\right)$ and $\beta'\left(t_0\right)$ are zero.

(3) Compute the first five terms of the series expansion at zero of the solution of the Legendre's equation $\left[1-(t)^2\right]y''-2ty'+\alpha(\alpha+1)y=0$, where $\alpha$ is a constant and can you guess the general term of the coefficient of the solution.

5 Answer any **two** : **2×7=14**

(1) (i) Find $L^{-1}\left(\dfrac{1}{z\left(z^2+4\right)^2}\right)$ and

(ii) Find $L(\cos ct)$.

(2) (i) Define second shifting theorem and

(ii) Find $L\left(e^{ct}\right)(z)$ using definition of Laplace Transform.

(3) Solve $y''-y'-2y=60e^t\sin 2t$ with $y=0$ and $y'=0$ when $t=0$ using Laplace Transform.

(4) State and prove Laplace Transform of Integral.

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